Tax competition between Regional Governments and National and Interregional economic growth

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Abstract

The purpose of this study is to examine a well-mentioned but rarely properly examined issue related to interregional tax competition and regional and national economic growth. We build an inter-regional neoclassical economic growth with regional governments' competition in taxation. We extend Solow-Uzawa's neoclassical growth model to any number of regions. The model treats wealth/capital accumulation, economic structures, factor distributions, interregional population distribution, amenity, regions' tax rates as endogenous variables. Firms' behavior is described by profit maximization, households' behavior by utility maximization, markets by perfect competition, and regional governments' behavior by choosing tax rates to maximize utility. We identify the existence of an equilibrium point and conduct comparative analysis to show how changes in, for instance, the utility elasticity of public goods, technologies, propensity to consume housing, and propensity to save, affect the long-run economic growth and structure.

Keywords: *tax competition; Nash equilibrium; multi-region economic dynamics; factor distribution; regional disparities in wealth and income; wealth accumulation; amenity.*

1. Introduction

This paper is concerned with the role of tax competition in interregional and national economic growth. The subject has been an important topic in the regional public economics literature (e.g., Wilson, 1986; Zodrow and Mieszkowski, 1986; Wildasin, 1988; Andersson and Forslid, 2003, Baldwin and Krugman, 2004, Bayindir-Upmann and Ziad, 2005, Borck and Pflüger, 2006; Ihori and Yang, 2009). This paper is to re-address the subject, with the usual neoclassical assumptions on the production technology and tax competition, but an alternative approach to behavior of household. The attempt is made to deal with the issue in a general equilibrium framework of a multi-regional economy, in which wealth accumulation is endogenous, capital and labor factors are freely mobile, and local public goods and local amenities are endogenous. We consider a Nash game in tax rates played between multiple regions with endogenous wealth accumulation.

Another issue we address is how strongly tax rates in different regions are related to rapid agglomeration observed in modern economies. Regional agglomeration has become increasingly more pronounced in different parts of the world. More and more people are moving to and living in a few metropolitan areas of the world are attracting more people. Regional agglomeration has caused attention of economists (Myrdal,

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1957; Hirschman, 1958; Kuznets, 1966; Bairoch, 1993). Earlier literature emphasizes dynamic interactions between industrial growth and the geographical concentration of industry. The contemporary literature on economic geography and economic development builds mathematical models to describe these dynamic processes (e.g., Krugman, 1991, Fujita et al, 1999, Forslid and Ottaviano, 2003, Zhang, 2008). Most of models in economic geography don't consider capital accumulation and tax competition. A unique contribution of this paper is to introduce tax competition into a neoclassical growth model with spatial agglomeration. concentrated on the study of interregional development with capital accumulation, taking account of factors such as environment and regional economic structure.

This study follows the neoclassical growth theory with wealth accumulation and amenity in regional growth and agglomeration. Although the so-called new economic geography (e.g., Krugman, 1993; Charlot, 2006; Bertoletti and Etro, 2015; Nocco, *et. al*., 2017; Parenti, *et.al*., 2017; and Picard and Zenou, 2018). In almost all the dynamic models of the new economic geography, regional amenities do not play a significant role in determining land rent and population mobility. To explain spatial economic agglomeration without taking account of spatial amenities and wealth accumulation may result in misleading results. Our study attempts to introduce wealth and amenity to spatial agglomeration theory. Moreover, as Tabuchi (2014: 50) observes, "The scopes of most of the theoretical studies published thus far have been limited to two regions in order for researchers to reach meaningful analytical results. The two-region NEG models tend to demonstrate that spatial distribution is dispersed in the early period (high trade costs or low manufacturing share) and agglomerated in one of the two regions in the late period (low trade costs or high manufacturing share). However, the two-region NEG models are too simple to describe the spatial distribution of economic activities in real-world economies. Since there are only two regions, their geographical locations are necessarily symmetric, and thus diverse spatial distributions cannot occur." Our model is developed for any number of regions.

Amenity is another important aspect of spatial agglomeration (e.g., Graves 1979; Roback 1982; Glaeser et. al. 2001; Partridge et al. 2008; Chen *et al.* 2013; Liu, et al. 2018; Tivadar and Jaye 2019). Zhang (1993) first introduced spatial amenity into utility in a general equilibrium framework. Zhang (1996) introduced spatial amenity into a formal regional growth model. The concept is an aggregated variable which is related to public services, local transportation systems, accessibilities, pollution, and human relations such as discrimination, and other factors. This study incorporates amenity into the consumer location decision by assuming that amenity is an endogenous variable. This paper is an extension of Zhang's previous models on interregional economic dynamics (Zhang 2009, 2018). The main extension is the introduction of local governments' tax competition with gaming approach. This rest of the paper is organized as follows. Section 2 develops the multi-regional model with capital accumulation, economic structure, and regional governments' tax competition. Section 3 identifies the existence of an equilibrium point. Section 4 carries out comparative static analysis with regards to the total factor productivities of the two sectors, the utility elasticity of public goods, the propensity to save, the propensity to consume housing, and the relative amenity. Section 5 concludes the study. Some of the results of section 3 are proved in the Appendix.

2. The Multi-regional Growth Model with Tax Competition

This paper is based on Zhang's dynamic interregional growth models (Zhang 1996, 2018). The main concern is to introduce tax competition (e.g., Wilson 1991) into the neoclassical growth model. The economy is composed of multiple regions and each region has a regional government. The sole role of the regional government is to provide public goods. Each region's financial resource is due to taxing the region's households. Households move freely between regions. Regional governments are competitive in taxation as they want to attract people by providing better public goods, but they cannot tax households arbitrarily as people can freely move between regions. There is competition in taxation between regions. The formal neoclassical trade theory is a mainstream in economic theory with a long and complicated history (Uzawa 1961; Oniki and Uzawa 1965; Brecher *et. al.* 2002; Sorger 2003; Zhang 2009). Following Uzawa (1961) and Zhang (1996), we consider that each region produces one goods and services. Most aspects of the two sectors are similar to the Uzawa two-sector growth model. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; the available factors are fully utilized at every moment. Households undertake saving, which implies that all earnings of firms are distributed in the form of payments to factors of production. Firms use all savings volunteered by households.

The national economy consists of J regions, indexed by $j = 1, \ldots, J$. All the markets within each region and between regions are perfectly competitive. There are no barrier and transaction costs for trade in commodities. No transportation cost and free trade implies equal price of the commodity in all the regions. Services are consumed in the region where they are supplied. All prices are measured in terms of the commodity whose price is unity. We use $w_j(t)$ and $r_j(t)$ to denote wage and interest respectively in the jth region. The interest rate is equalized throughout the national economy, i.e., $r(t) = r_j(t)$. The population N is constant and homogenous. People move freely without any transaction costs between regions, choosing residential location, consumption bundles, and saving to maximize utility. Region j is endowed with fixed homogenous land L_j solely available for residential use. Each region's residential condition is described by an aggregated variable, amenity. As amenity and land are immobile, wage rates and land rent vary between regions. We use subscripts, i, s, to stand for the capital goods and services sectors, respectively. Let $F_{jq}(t)$stand for the output levels of q's sector in region j at time t, $q = i, s$

Behavior of producers

Firms in each region employ capital $K_{jq}(t)$ and labor $N_{jq}(t)$ to supply with the following production functions:

$$F_{jq}(t) = A_{jq} K_{jq}^{\alpha_{jq}}(t) N_{jq}^{\beta_{jq}}(t), \ j = 1, \ldots, J, \ q = i, s. \quad (1)$$

We use $p_j(t)$ to stand for region j's services price. The marginal conditions imply:

$$r(t) + \delta_{kj} = \frac{\alpha_{ji} F_{ji}(t)}{K_{ji}(t)} = \frac{\alpha_{js} p_j(t) F_{js}(t)}{K_{js}(t)}, \quad w_j(t) = \frac{\beta_{ji} F_{ji}(t)}{N_{ji}(t)} = \frac{\beta_{js} p_j(t) F_{js}(t)}{N_{js}(t)}, \quad (2)$$

where δ_{kj} are depreciation rates of physical capital in region j. It should be noted that this study does not consider possible externalities on production due to production agglomeration and public goods and taxation on firms located in each region. It is important to take account of tax rates on production sides. As analysis is too complicated, this study is limited to the case that only consumers are subject to taxation. How to introduce taxes on production sides with tax competition is referred to, for instance, Wilson (1991) and Saez and Stantcheva (2018).

The household's current income, disposable income, and budget

In order to define incomes, it is necessary to determine land ownership structure. Land properties may be distributed in multiple ways under various institutions. To simplify the model, we accept the assumption of absent landownership, which means that the income of land rent is spent outside the economic system. A possible reasoning for this that the land is owned by the government, people can rent the land in competitive market, and the government uses the income for military or other public purposes. Consumers make decisions on choice of lot size, consumption levels of services and commodities as well as on how much to save. We describe behavior of households by Zhang's model (e.g., Zhang, 1993, 2005, 2008).

Let $\bar{k}_j(t)$ stand for the per household wealth in region j. Region $j's$ representative household has the following current income:

$$y_j(t) = \tau_j^k(t)\, r(t)\, \bar{k}_j(t) + \tau_j^w(t)\, w_j(t), \quad (3)$$

where $\tau_j^k(t)$ and $\tau_j^w(t)$ are respectively defined as follows:

$$\tau_j^x(t) = 1 - \bar{\tau}_j^x\, \tau_j(t), \; x = k, w. \quad (4)$$

The terms $\bar{\tau}_j^k\, \tau_j(t)$ and $\bar{\tau}_j^w\, \tau_j(t)$ are respectively the tax rates on wealth income and wage in region j. For simplicity of analysis, we assume a constant co-relation between the tax rates in the same region, which are measured by two parameters, $\bar{\tau}_j^k$ and $\bar{\tau}_j^w$.

Hence, region $j's$ government determines a single variable $\tau_j(t)$ when making its optimal decision. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income is given by:

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \ (5)$$

The disposable income is used for saving and consumption. The value of wealth $\bar{k}_j(t)$ is a flow variable. At each point in time, the household distributes the total available budget between housing $l_j(t)$, saving $s_j(t)$, consumption of goods $c_{ji}(t)$, and consumption of goods $c_{js}(t)$. The budget constraint is thus as follows:

$$R_j(t)\, l_j(t) + c_{ji}(t) + p_j(t)\, c_{js}(t) + s_j(t) = \hat{y}_j(t), \ (6)$$

where $R_j(t)$ is region j's land rent.

Utility, amenity and optimal behavior of the household

We assume that utility level $U_j(t)$ that the consumer obtains is dependent on the consumption levels of lot size, commodity, services, and saving. The utility level of the typical consumer in region j is:

$$U_j(t) = \theta_j(t)\, l_j^{\eta_0}(t)\, c_{ji}^{\xi_0}\, c_{js}^{\gamma_0}(t)\, s_j^{\lambda_0}(t), \ \eta_0, \gamma_0, \xi_0, \lambda_0 > 0, \ (7)$$

in which η_0, ξ_0, γ_0, and λ_0 are the representative household's elasticity of utility with regard to lot size, commodity, services, and savings in region j. We call η_0, ξ_0, γ_0, and λ_0 propensities to consume lot size, goods, and services, and to hold wealth (save), respectively. In (7), $\theta_j(t)$ is region j's amenity level. Amenities are affected by infrastructures, regional cultures and climates (e.g., Kanemoto, 1980; Diamond and Tolley, 1981; Blomquist, et al. 1988). In this study, we assume that amenity is affected by population and local public goods $G_j(t)$. We specify $\theta_j(t)$ as follows:

$$\theta_j(t) = \bar{\theta}_j\, N_j^{d}(t)\, G_j^{\bar{d}}(t), \ j = 1, \ldots, J, \ (8)$$

where $\bar{\theta}_j (> 0)$, d, $\bar{d} > 0$, are parameters and $N_j(t)$ is region j's population. We don't specify signs of d as population may have either positive or negative effects on regional attractiveness. As Chen *et al.* (2013: 269) observe: "The presence of both positive and negative population externalities suggests that the steady state (or competitive) pattern may differ from an optimal pattern in which all the external benefits and costs of households' migration decisions are internalized." It should be noted that through amenity we relate our approach to hedonic price modelling (e.g., Rosen, 1974; Helbich *et al.*, 2014). As shown late on we make it possible to relate environment and housing prices (Dubin, 1992; Can and Megbolugbe, 1997; Sheppard, 1997; Malpezzi, 2003; McMillen, 2010; and Ahlfeldt, 2011).

Maximizing $U_j(t)$ subject to the budget constraints (5) yields:

$$l_j(t)\, R_j(t) = \eta\, \hat{y}_j(t), \;\; c_{ji}(t) = \xi\, \hat{y}_j(t), \;\; p_j(t)\, c_{js}(t) = \gamma\, \hat{y}_j(t), \;\; s_j(t) = \lambda\, \hat{y}_j(t), \;\; (9)$$

where

$$\eta \equiv \eta_0\, \rho, \;\; \xi \equiv \xi_0\, \rho, \;\; \gamma \equiv \gamma_0\, \rho, \;\; \lambda \equiv \lambda_0\, \rho, \;\; \rho \equiv \frac{1}{\eta_0 + \xi_0 + \gamma_0 + \lambda_0}.$$

Wealth accumulation

According to the definitions of $s_j(t)$, the wealth accumulation of the representative household in region j is given by:

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{10}$$

Equalization of utility levels between regions

As households are freely mobile between the regions, the utility level of people should be equal, irrespective of in which region they live, i.e.

$$U_j(t) = U_q(t), \;\; j, q = 1, \ldots, J. \;\; (11)$$

We don't take account of possible costs for migration. Taking account of changes in houses makes it difficult to model the behavior of households. Wage equalization between regions is often used as the equilibrium mechanism of population mobility over space.

Demand and supply balances

The total capital stocks $K(t)$ employed by the production sectors is equal to the total wealth owned by the households of all the regions. That is

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{k}_j(t) N_j(t), \quad (12)$$

in which $K_j(t) \equiv K_{ji}(t) + K_{js}(t)$.

A region's supply of services is consumed by the region

$$c_{js}(t) N_j(t) = F_{js}(t). \quad (13)$$

Full employment of input factors

Each region's land is fully occupied by the region's population:

$$l_j(t) N_j(t) = L_j. \quad (14)$$

The assumption that labor force and land are fully employed is represented by

$$N_{ji}(t) + N_{js}(t) = N_j(t), \; j = 1, \dots, J, \quad (15)$$

$$\sum_{j=1}^{J} N_j(t) = N. \quad (16)$$

The regional governments' budgets

As the tax income is solely spent on supplying public goods, we have the regional government budgets as follows:

$$\bar{Y}_j(t) = Y_j^0(t) \tau_j(t) N_j(t), \quad (17)$$

where

$$Y_j^0(t) \equiv \bar{\tau}_j^k \, r(t) \, \bar{k}_j(t) + \bar{\tau}_j^w w(t).$$

We assume that all the tax income is spent on public goods and the public sector receives no financial support from any other source. We thus have:

$$G_j(t) = \bar{Y}_j(t). \quad (18)$$

Interregional tax competition

The local governments play a Nash equilibrium game in tax rates. We consider that each regional government maximizes the utility level that the representative household receives by choosing living and working in the region. From (7)-(9), we can write utility function as follows:

$$U_j\left(t,\left(\tau_j(t)\right)\right) = \bar{\theta}_j \, L_j^{\eta_0} \, \xi^{\xi_0} \, \gamma^{\gamma_0} \, \lambda^{\lambda_0} \, N_j^{d-\eta_0}(t) \, G_j^{\bar{d}}(t) \, p_j^{-\gamma_0}(t) \, \hat{y}_j^{\tilde{\rho}}(t). \quad (19)$$

where we use (14) and

$$\tilde{\rho} = \gamma_0 + \xi_0 + \lambda_0.$$

Insert (15) and (16) in (17)

$$U_j\left(t,\left(\tau_j(t)\right)\right) = u_j(t) \, \tau_j^{\bar{d}}(t) \, \hat{y}_j^{\tilde{\rho}}(t), \quad (20)$$

where

$$u_j(t) \equiv \bar{\theta}_j \, L_j^{\eta_0} \, \xi^{\xi_0} \, \gamma^{\gamma_0} \, \lambda^{\lambda_0} \, N_j^{d+\bar{d}-\eta_0}(t) \, Y_j^{0\bar{d}}(t) \, p_j^{-\gamma_0}(t).$$

From (5), we have

$$\hat{y}_j(t) = h_j(t) - Y_j^0(t) \, \tau_j(t), \quad (21)$$

where

$$h_j(t) \equiv \bar{r}(t) + \bar{k}_j(t) + r(t)\,\bar{k}_j(t) + w_j(t).$$

Insert (21) in (20)

$$U_j\left(t,\left(\tau_m(t)\right)\right) = u_j(t)\,\tau_j^{\bar{d}}(t)\left(h_j(t) - Y_j^0(t)\,\tau_j(t)\right)^{\tilde{\rho}}. \quad (22)$$

Region j' government maximizes $U_j\left(t,\left(\tau_m(t)\right)\right)$ by choosing $\tau_j(t)$ with all the other region governments' $\tau_m(t)$ as given. Taking the partial derivative of $U_j\left(t,\left(\tau_m(t)\right)\right)$ in $\tau_j(t)$ and the applying the marginal condition yields:

$$\tau_j(t) = \frac{\tilde{\mathrm{d}}\,h_j(t)}{Y_j^0(t)}, \quad (23)$$

where $\tilde{\mathrm{d}} = \bar{d}/(\tilde{\rho} + \bar{d})$. We have an equilibrium in government game when each government chooses its optimal tax rates, given the tax rates chosen by all the other governments. It should be noted that one might also consider some other strategic variables for the governments. For instance, Wildasin (1986, 1991) considers public expenditure levels as the strategic variables.

We have thus built the interregional growth model with endogenous capital accumulation, regional capital and labor distribution in a perfectly competitive economy with the government intervention.

3. Equilibrium Point

As shown in the Appendix, it is difficult to make a genuine dynamic analysis of the nonlinear system. For illustration, the rest of the study simulates the model. This section identifies the existence of an equilibrium point. The procedure to determine the equilibrium point is provided in the Appendix. We analyze equilibrium structure for a 3-region economy. We specify parameter values as follows:

$$N = 100,\ \lambda_0 = 0.75,\ \xi_0 = 0.1,\ \eta_0 = 0.07,\ \gamma_0 = 0.07,\ d = -0.05,\ \bar{d} = 0.1;$$

$$\begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{3i} \end{pmatrix} = \begin{pmatrix} 1.3 \\ 1.2 \\ 1.1 \end{pmatrix}, \begin{pmatrix} A_{1s} \\ A_{2s} \\ A_{3s} \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1.1 \\ 1 \end{pmatrix}, \begin{pmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{pmatrix} = \begin{pmatrix} 0.32 \\ 0.31 \\ 0.3 \end{pmatrix}, \begin{pmatrix} \alpha_{1s} \\ \alpha_{2s} \\ \alpha_{3s} \end{pmatrix} = \begin{pmatrix} 0.32 \\ 0.32 \\ 0.32 \end{pmatrix},$$
$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \\ 20 \end{pmatrix}, \begin{pmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \\ \bar{\theta}_3 \end{pmatrix} = \begin{pmatrix} 3.8 \\ 4.2 \\ 5 \end{pmatrix}, \begin{pmatrix} \bar{\tau}_{1k} \\ \bar{\tau}_{2k} \\ \bar{\tau}_{3k} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.7 \\ 0.8 \end{pmatrix}, \begin{pmatrix} \bar{\tau}_{1w} \\ \bar{\tau}_{2w} \\ \bar{\tau}_{3w} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \delta_{k1} \\ \delta_{k2} \\ \delta_{k3} \end{pmatrix} = \begin{pmatrix} 0.04 \\ 0.05 \\ 0.06 \end{pmatrix}. \tag{24}$$

The population is fixed at 100. The propensities to save, to consume commodity, to consume housing, and to consume lot size are respectively 0.75, 0.1, 0.07 and 0.07. The utility elasticity for the population is negative. The utility elasticity for public goods is 0.1.

Region 1's levels of productivity of the two sectors are highest; region 2's levels are the next; and region 3's levels of productivity of the two sectors are lowest. We specify values of α_{jk} close to 0.3. With regard to the technological parameters, for illustration what are important in our interregional study are their relative values. The presumed productivity differences between the regions are not very large. We have the regional GDPs and national GDP as follows:

$$Y_j = F_{ji} + p_j F_{js}, \quad Y = Y_1 + Y_2 + Y_3.$$

The simulation confirms that the system has an equilibrium point. We list the equilibrium values in (25):

$$Y = 183.96, \quad K = 361.4, \quad r = 0.115,$$

$$\begin{pmatrix}\tau_{1k}\\ \tau_{2k}\\ \tau_{3k}\end{pmatrix} = \begin{pmatrix}0.198\\ 0.27\\ 0.302\end{pmatrix}, \begin{pmatrix}\tau_{1w}\\ \tau_{2w}\\ \tau_{3w}\end{pmatrix} = \begin{pmatrix}0.395\\ 0.386\\ 0.377\end{pmatrix}, \begin{pmatrix}\bar{Y}_1\\ \bar{Y}_2\\ \bar{Y}_3\end{pmatrix} = \begin{pmatrix}30.9\\ 10.1\\ 10.9\end{pmatrix}, \begin{pmatrix}Y_1\\ Y_2\\ Y_3\end{pmatrix} = \begin{pmatrix}110.5\\ 35.5\\ 37.9\end{pmatrix},$$

$$\begin{pmatrix}N_1\\ N_2\\ N_3\end{pmatrix} = \begin{pmatrix}53.4\\ 20.5\\ 26.1\end{pmatrix}, \begin{pmatrix}K_1\\ K_2\\ K_3\end{pmatrix} = \begin{pmatrix}228.4\\ 67.1\\ 65.9\end{pmatrix}, \begin{pmatrix}\bar{K}_1\\ \bar{K}_2\\ \bar{K}_3\end{pmatrix} = \begin{pmatrix}212.6\\ 70.1\\ 75.8\end{pmatrix}, \begin{pmatrix}F_{1i}\\ F_{2i}\\ F_{3i}\end{pmatrix} = \begin{pmatrix}90.4\\ 29\\ 30.1\end{pmatrix},$$

$$\begin{pmatrix}F_{1s}\\ F_{2s}\\ F_{3s}\end{pmatrix} = \begin{pmatrix}18.6\\ 6.06\\ 6.55\end{pmatrix}, \begin{pmatrix}N_{1i}\\ N_{2i}\\ N_{3i}\end{pmatrix} = \begin{pmatrix}43.68\\ 16.74\\ 21.38\end{pmatrix}, \begin{pmatrix}N_{1s}\\ N_{2s}\\ N_{3s}\end{pmatrix} = \begin{pmatrix}9.72\\ 3.73\\ 4.76\end{pmatrix}, \begin{pmatrix}K_{1i}\\ K_{2i}\\ K_{3i}\end{pmatrix} = \begin{pmatrix}187.8\\ 54.4\\ 53\end{pmatrix},$$

$$\begin{pmatrix}K_{1s}\\ K_{2s}\\ K_{3s}\end{pmatrix} = \begin{pmatrix}41.6\\ 12.7\\ 12.9\end{pmatrix}, \begin{pmatrix}p_1\\ p_2\\ p_3\end{pmatrix} = \begin{pmatrix}1.083\\ 1.078\\ 1.079\end{pmatrix}, \begin{pmatrix}w_1\\ w_2\\ w_3\end{pmatrix} = \begin{pmatrix}1.41\\ 1.19\\ 1.01\end{pmatrix}, \begin{pmatrix}R_1\\ R_2\\ R_3\end{pmatrix} = \begin{pmatrix}2.01\\ 0.33\\ 0.35\end{pmatrix},$$

$$\begin{pmatrix}\theta_1\\ \theta_2\\ \theta_3\end{pmatrix} = \begin{pmatrix}4.39\\ 4.55\\ 5.39\end{pmatrix}, \begin{pmatrix}\bar{k}_1\\ \bar{k}_1\\ \bar{k}_1\end{pmatrix} = \begin{pmatrix}4.04\\ 3.42\\ 2.9\end{pmatrix}, \begin{pmatrix}l_1\\ l_2\\ l_3\end{pmatrix} = \begin{pmatrix}0.19\\ 0.98\\ 0.77\end{pmatrix}, \begin{pmatrix}c_{1i}\\ c_{2i}\\ c_{3i}\end{pmatrix} = \begin{pmatrix}0.54\\ 0.46\\ 0.39\end{pmatrix},$$

$$\begin{pmatrix}c_{1s}\\ c_{2s}\\ c_{3s}\end{pmatrix} = \begin{pmatrix}0.35\\ 0.3\\ 0.25\end{pmatrix}. \quad (25)$$

Region 1's tax rate on wage is highest, but region 1's tax rate on wealth income is lowest. The household in region 1 has consumption levels of goods and services, highest wage rate, and wealth. The land rent in region 1 is highest and lot size and amenity are lowest. Region1 provides highest level of public goods. More than half of the population is attracted to region 1. Region 1 employs more capital stocks that the capital stocks owned by the region's population.

4. Comparative dynamic analysis

We found the equilibrium point in the previous sector. This section shows how the economic system reacts to changes in different parameters. Following the Lemma, we can give the equilibrium values of all the variables. In the rest of this study we use $\bar{\Delta} x_j$ to stand for the change rate of the variable x_j in percentage due to changes in a parameter value.

4.1. The utility elasticity for public goods is increased

We first examine how the equilibrium structure is affected by the following rise in the utility elasticity for public goods: $\bar{\mathrm{d}} = 0.1 \text{ to } 0.102$. We list the simulation result in (26):

$$\bar{\Delta} Y = -0.95, \ \bar{\Delta} K = -3.64, \ \bar{\Delta} r = 4.15,$$

$$\begin{pmatrix} \bar{\Delta}\tau_{1k} \\ \bar{\Delta}\tau_{2k} \\ \bar{\Delta}\tau_{3k} \end{pmatrix} = \begin{pmatrix} \bar{\Delta}\tau_{1w} \\ \bar{\Delta}\tau_{2w} \\ \bar{\Delta}\tau_{3w} \end{pmatrix} = \begin{pmatrix} 2.4 \\ 2.2 \\ 2 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{Y}_1 \\ \bar{\Delta}\bar{Y}_2 \\ \bar{\Delta}\bar{Y}_3 \end{pmatrix} = \begin{pmatrix} 4.4 \\ -4.4 \\ -0.8 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}Y_1 \\ \bar{\Delta}Y_2 \\ \bar{\Delta}Y_3 \end{pmatrix} = \begin{pmatrix} 1.7 \\ -6.9 \\ -3.1 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_1 \\ \bar{\Delta}N_2 \\ \bar{\Delta}N_3 \end{pmatrix} = \begin{pmatrix} 3.2 \\ -5.7 \\ -2 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_1 \\ \bar{\Delta}K_2 \\ \bar{\Delta}K_3 \end{pmatrix} = \begin{pmatrix} -1.3 \\ -9.6 \\ -5.7 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{K}_1 \\ \bar{\Delta}\bar{K}_2 \\ \bar{\Delta}\bar{K}_3 \end{pmatrix} = \begin{pmatrix} 2.3 \\ -6.5 \\ -2.8 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}F_{1i} \\ \bar{\Delta}F_{2i} \\ \bar{\Delta}F_{3i} \end{pmatrix} = \begin{pmatrix} 1.6 \\ -7 \\ -3.2 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}F_{1s} \\ \bar{\Delta}F_{2s} \\ \bar{\Delta}F_{3s} \end{pmatrix} = \begin{pmatrix} 2.3 \\ -6.5 \\ -2.8 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}N_{1i} \\ \bar{\Delta}N_{2i} \\ \bar{\Delta}N_{3i} \end{pmatrix} = \begin{pmatrix} 3 \\ -5.8 \\ -2.1 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}N_{1s} \\ \bar{\Delta}N_{2s} \\ \bar{\Delta}N_{3s} \end{pmatrix} = \begin{pmatrix} 3.8 \\ -5.3 \\ -1.6 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_{1i} \\ \bar{\Delta}K_{2i} \\ \bar{\Delta}K_{3i} \end{pmatrix} = \begin{pmatrix} -1.5 \\ -9.7 \\ -5.8 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}K_{1s} \\ \bar{\Delta}K_{2s} \\ \bar{\Delta}K_{3s} \end{pmatrix} = \begin{pmatrix} -0.8 \\ -9.1 \\ -5.3 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}p_1 \\ \bar{\Delta}p_2 \\ \bar{\Delta}p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.04 \\ 0.08 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}w_1 \\ \bar{\Delta}w_2 \\ \bar{\Delta}w_3 \end{pmatrix} = \begin{pmatrix} -1.4 \\ -1.3 \\ -1.2 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}R_1 \\ \bar{\Delta}R_2 \\ \bar{\Delta}R_3 \end{pmatrix} = \begin{pmatrix} 2.3 \\ -6.9 \\ -2.7 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}\theta_1 \\ \bar{\Delta}\theta_2 \\ \bar{\Delta}\theta_3 \end{pmatrix} = \begin{pmatrix} 0.97 \\ 0.28 \\ 0.5 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \end{pmatrix} = \begin{pmatrix} -0.84 \\ -0.8 \\ -0.77 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}l_1 \\ \bar{\Delta}l_2 \\ \bar{\Delta}l_3 \end{pmatrix} = \begin{pmatrix} -3.1 \\ 6.1 \\ 2 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}c_{1i} \\ \bar{\Delta}c_{2i} \\ \bar{\Delta}c_{3i} \end{pmatrix} = \begin{pmatrix} -0.85 \\ -0.78 \\ -0.73 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}c_{1s} \\ \bar{\Delta}c_{2s} \\ \bar{\Delta}c_{3s} \end{pmatrix} = \begin{pmatrix} -0.85 \\ -0.83 \\ -0.81 \end{pmatrix}. \quad (26)$$

The positive utility elasticity for public goods implies that more public goods in one region will makes the region more attractive. Competition between the regional governments leads in that all the regions increase their tax rates. Region 1 attracts more people, while the other two regions lose people. As region has is more advanced than the other two regions in technology, the rise in elasticity implies that public investment is relatively more important than the other factors in amenity functions. Although it increases its tax rates more than the other regions, region 1 becomes more attractive as its tax income and public goods increases are more than the other two regions. The national capital and income are reduced in association with rises in the rate of interest. The prices of services are slightly increased. All the households have lower consumption levels of goods and services and less wealth. The amenity levels in all the regions are enhanced. The residential rent (lot size) is enhanced (reduced) in region 1 and reduced (lot size) in the other two regions.

4.2. The amenity elasticity for the regional population is increased

We study how the equilibrium structure is affected by the following rise in the amenity elasticity for the regional population: $\mathrm{d} = -0.05 \text{ to} - 0.04.$ We list the simulation result in (27):

$$\bar{\Delta}Y = 3.39, \bar{\Delta}K = 3.81, \bar{\Delta}r = 1.99,$$

$$\begin{pmatrix} \bar{\Delta}\tau_{1k} \\ \bar{\Delta}\tau_{2k} \\ \bar{\Delta}\tau_{3k} \end{pmatrix} = \begin{pmatrix} \bar{\Delta}\tau_{1w} \\ \bar{\Delta}\tau_{2w} \\ \bar{\Delta}\tau_{3w} \end{pmatrix} = \begin{pmatrix} 0.55 \\ 0.49 \\ 0.45 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{Y}_1 \\ \bar{\Delta}\bar{Y}_2 \\ \bar{\Delta}\bar{Y}_3 \end{pmatrix} = \begin{pmatrix} 30.8 \\ -46.2 \\ -26.8 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}Y_1 \\ \bar{\Delta}Y_2 \\ \bar{\Delta}Y_3 \end{pmatrix} = \begin{pmatrix} 29.9 \\ -46.5 \\ -27.3 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta}N_1 \\ \bar{\Delta}N_2 \\ \bar{\Delta}N_3 \end{pmatrix} = \begin{pmatrix} 30.8 \\ -46.2 \\ -26.8 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_1 \\ \bar{\Delta}K_2 \\ \bar{\Delta}K_3 \end{pmatrix} = \begin{pmatrix} 28 \\ -47.2 \\ -28.2 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{K}_1 \\ \bar{\Delta}\bar{K}_2 \\ \bar{\Delta}\bar{K}_3 \end{pmatrix} = \begin{pmatrix} 30.8 \\ -46.2 \\ -26.8 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta}F_{1i} \\ \bar{\Delta}F_{2i} \\ \bar{\Delta}F_{3i} \end{pmatrix} = \begin{pmatrix} 29.7 \\ -46.6 \\ -27.4 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}F_{1s} \\ \bar{\Delta}F_{2s} \\ \bar{\Delta}F_{3s} \end{pmatrix} = \begin{pmatrix} 30.8 \\ -46.2 \\ -26.8 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}N_{1i} \\ \bar{\Delta}N_{2i} \\ \bar{\Delta}N_{3i} \end{pmatrix} = \begin{pmatrix} 30.6 \\ -46.3 \\ -27 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta}N_{1s} \\ \bar{\Delta}N_{2s} \\ \bar{\Delta}N_{3s} \end{pmatrix} = \begin{pmatrix} 31.7 \\ -45.8 \\ -26.4 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_{1i} \\ \bar{\Delta}K_{2i} \\ \bar{\Delta}K_{3i} \end{pmatrix} = \begin{pmatrix} 28.8 \\ -47.3 \\ -28.3 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_{1s} \\ \bar{\Delta}K_{2s} \\ \bar{\Delta}K_{3s} \end{pmatrix} = \begin{pmatrix} 28.9 \\ -46.9 \\ -27.7 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta}p_1 \\ \bar{\Delta}p_2 \\ \bar{\Delta}p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.02 \\ 0.04 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}w_1 \\ \bar{\Delta}w_2 \\ \bar{\Delta}w_3 \end{pmatrix} = \begin{pmatrix} -0.69 \\ -0.62 \\ -0.56 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}R_1 \\ \bar{\Delta}R_2 \\ \bar{\Delta}R_3 \end{pmatrix} = \begin{pmatrix} 30.8 \\ -46.2 \\ -26.8 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta}\theta_1 \\ \bar{\Delta}\theta_2 \\ \bar{\Delta}\theta_3 \end{pmatrix} = \begin{pmatrix} 5.8 \\ -0.7 \\ 1.4 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \end{pmatrix} = \begin{pmatrix} -0.03 \\ 0.06 \\ 0.1 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}l_1 \\ \bar{\Delta}l_2 \\ \bar{\Delta}l_3 \end{pmatrix} = \begin{pmatrix} -23.6 \\ 85.8 \\ 36.7 \end{pmatrix},$$
$$\begin{pmatrix} \bar{\Delta}c_{1i} \\ \bar{\Delta}c_{2i} \\ \bar{\Delta}c_{3i} \end{pmatrix} = \begin{pmatrix} -0.03 \\ 0.04 \\ 0.1 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}c_{1s} \\ \bar{\Delta}c_{2s} \\ \bar{\Delta}c_{3s} \end{pmatrix} = \begin{pmatrix} -0.03 \\ 0.02 \\ 0.06 \end{pmatrix}. \quad (27)$$

As the population influence on amenity is weakened, the population is more concentrated in region 1. The national development is encouraged. The national output and capital are enhanced. The rate of interest is reduced. The tax rates of all the regions are enhanced. Region 1 supplies more public goods and the other two regions' public goods are reduced. The other macroeconomic real variables in the three regions are affected similarly. Region 1 has higher levels of the total output, the output levels of the two sectors, and capital stocks and labor force employed by each sector. The same variables of the other two regions are reduced. The service prices are slightly increased. The wage rates are reduced. Region 2's amenity is reduced. The other two regions' amenity levels are enhanced. Region 1 has lower levels of consumption of goods and services. The other two regions' representative households consume more goods and services. Region 1's household pays higher rent and lives in smaller house.

4.3. Region 3's residential land is expanded

We study how the equilibrium structure is affected by the following expansion in region 3's residential land: $L_3 = 20 \text{ to } 21$. We list the simulation result in (28):

$$\bar{\Delta} Y = -0.75, \ \bar{\Delta} K = -0.83, \ \bar{\Delta} r = -0.42,$$

$$\begin{pmatrix} \bar{\Delta}\tau_{1k} \\ \bar{\Delta}\tau_{2k} \\ \bar{\Delta}\tau_{3k} \end{pmatrix} = \begin{pmatrix} \bar{\Delta}\tau_{1w} \\ \bar{\Delta}\tau_{2w} \\ \bar{\Delta}\tau_{3w} \end{pmatrix} = \begin{pmatrix} -0.12 \\ -0.1 \\ -0.09 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{Y}_1 \\ \bar{\Delta}\bar{Y}_2 \\ \bar{\Delta}\bar{Y}_3 \end{pmatrix} = \begin{pmatrix} -4.09 \\ -4.83 \\ 12.12 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}Y_1 \\ \bar{\Delta}Y_2 \\ \bar{\Delta}Y_3 \end{pmatrix} = \begin{pmatrix} -3.95 \\ -4.7 \\ 12.28 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_1 \\ \bar{\Delta}N_2 \\ \bar{\Delta}N_3 \end{pmatrix} = \begin{pmatrix} -4.1 \\ -4.83 \\ 12.15 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_1 \\ \bar{\Delta}K_2 \\ \bar{\Delta}K_3 \end{pmatrix} = \begin{pmatrix} -3.65 \\ -4.42 \\ 12.59 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{K}_1 \\ \bar{\Delta}\bar{K}_2 \\ \bar{\Delta}\bar{K}_3 \end{pmatrix} = \begin{pmatrix} -4.09 \\ -4.83 \\ 12.12 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}F_{1i} \\ \bar{\Delta}F_{2i} \\ \bar{\Delta}F_{3i} \end{pmatrix} = \begin{pmatrix} -3.92 \\ -4.67 \\ 13.32 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}F_{1s} \\ \bar{\Delta}F_{2s} \\ \bar{\Delta}F_{3s} \end{pmatrix} = \begin{pmatrix} -4.09 \\ -4.83 \\ 12.13 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}N_{1i} \\ \bar{\Delta}N_{2i} \\ \bar{\Delta}N_{3i} \end{pmatrix} = \begin{pmatrix} -4.07 \\ -4.8 \\ 12.18 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_{1s} \\ \bar{\Delta}N_{2s} \\ \bar{\Delta}N_{3s} \end{pmatrix} = \begin{pmatrix} -4.23 \\ -4.96 \\ 11.99 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_{1i} \\ \bar{\Delta}K_{2i} \\ \bar{\Delta}K_{3i} \end{pmatrix} = \begin{pmatrix} -3.62 \\ -4.39 \\ 12.63 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_{1s} \\ \bar{\Delta}K_{2s} \\ \bar{\Delta}K_{3s} \end{pmatrix} = \begin{pmatrix} -3.79 \\ -4.55 \\ 12.44 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}p_1 \\ \bar{\Delta}p_2 \\ \bar{\Delta}p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -0.01 \\ -0.01 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}w_1 \\ \bar{\Delta}w_2 \\ \bar{\Delta}w_3 \end{pmatrix} = \begin{pmatrix} 0.15 \\ 0.13 \\ 0.12 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}R_1 \\ \bar{\Delta}R_2 \\ \bar{\Delta}R_3 \end{pmatrix} = \begin{pmatrix} -4.09 \\ -4.83 \\ 6.79 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\theta_1 \\ \bar{\Delta}\theta_2 \\ \bar{\Delta}\theta_3 \end{pmatrix} = \begin{pmatrix} -0.21 \\ -0.25 \\ 0.57 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \end{pmatrix} = \begin{pmatrix} 0.01 \\ -0.01 \\ -0.02 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}l_1 \\ \bar{\Delta}l_2 \\ \bar{\Delta}l_3 \end{pmatrix} = \begin{pmatrix} 4.27 \\ 5.01 \\ -6.37 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}c_{1i} \\ \bar{\Delta}c_{2i} \\ \bar{\Delta}c_{3i} \end{pmatrix} = \begin{pmatrix} 0.01 \\ -0.01 \\ -0.02 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}c_{1s} \\ \bar{\Delta}c_{2s} \\ \bar{\Delta}c_{3s} \end{pmatrix} = \begin{pmatrix} 0.01 \\ -0.01 \\ -0.01 \end{pmatrix}. \tag{28}$$

The enlarged residential area attracts more people to region 3. The increased population is associated with increases in region 3's expenditures on public goods. This makes the region more attractive. At the new equilibrium, region 3's lot size per household is reduced irrespective of the region's expansion of residential land. As more people move to the lest productive region, the nation has lower income and less capital stocks. The rate of interest is reduced. The wage rates are enhanced. The prices of services are reduced slightly. Region 3's household enjoys higher amenity but pays higher rent and lives in smaller house. Region 3's other real macro-economic variables are augmented, while the corresponding variables of the other two regions are reduced. The regions' governments reduce the tax rates.

4.4. Region 3's amenity parameter is increased

We study how the equilibrium structure is affected by the following rise in region 3's amenity parameter: $\bar{\theta}_3 = 5 \text{ to } 5.1$. We list the simulation result in (29):

$$\bar{\Delta} Y = -0.43, \ \bar{\Delta} K = -0.48, \ \bar{\Delta} r = -0.24,$$

$$\begin{pmatrix}\bar{\Delta}\tau_{1k}\\ \bar{\Delta}\tau_{2k}\\ \bar{\Delta}\tau_{3k}\end{pmatrix}=\begin{pmatrix}\bar{\Delta}\tau_{1w}\\ \bar{\Delta}\tau_{2w}\\ \bar{\Delta}\tau_{3w}\end{pmatrix}=\begin{pmatrix}-0.07\\ -0.06\\ -0.05\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}\bar{Y}_1\\ \bar{\Delta}\bar{Y}_2\\ \bar{\Delta}\bar{Y}_3\end{pmatrix}=\begin{pmatrix}-2.36\\ -2.79\\ 7\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}Y_1\\ \bar{\Delta}Y_2\\ \bar{\Delta}Y_3\end{pmatrix}=\begin{pmatrix}-2.28\\ -2.72\\ 7.09\end{pmatrix},$$

$$\begin{pmatrix}\bar{\Delta}N_1\\ \bar{\Delta}N_2\\ \bar{\Delta}N_3\end{pmatrix}=\begin{pmatrix}-2.36\\ -2.79\\ 7.01\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}K_1\\ \bar{\Delta}K_2\\ \bar{\Delta}K_3\end{pmatrix}=\begin{pmatrix}-2.1\\ -2.55\\ 7{,}26\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}\bar{K}_1\\ \bar{\Delta}\bar{K}_2\\ \bar{\Delta}\bar{K}_3\end{pmatrix}=\begin{pmatrix}-2.36\\ -2.79\\ 7\end{pmatrix},$$

$$\begin{pmatrix}\bar{\Delta}F_{1i}\\ \bar{\Delta}F_{2i}\\ \bar{\Delta}F_{3i}\end{pmatrix}=\begin{pmatrix}-2.26\\ -2{,}7\\ 7.11\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}F_{1s}\\ \bar{\Delta}F_{2s}\\ \bar{\Delta}F_{3s}\end{pmatrix}=\begin{pmatrix}-2.36\\ -2.79\\ 7\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}N_{1i}\\ \bar{\Delta}N_{2i}\\ \bar{\Delta}N_{3i}\end{pmatrix}=\begin{pmatrix}-2.35\\ -2.77\\ 7.03\end{pmatrix},$$

$$\begin{pmatrix}\bar{\Delta}N_{1s}\\ \bar{\Delta}N_{2s}\\ \bar{\Delta}N_{3s}\end{pmatrix}=\begin{pmatrix}-2.44\\ -2.87\\ 6.93\end{pmatrix},\begin{pmatrix}\bar{\Delta}K_{1i}\\ \bar{\Delta}K_{2i}\\ \bar{\Delta}K_{3i}\end{pmatrix}=\begin{pmatrix}-2.09\\ -2.53\\ 2.28\end{pmatrix},\begin{pmatrix}\bar{\Delta}K_{1s}\\ \bar{\Delta}K_{2s}\\ \bar{\Delta}K_{3s}\end{pmatrix}=\begin{pmatrix}-2{,}18\\ -2.63\\ 7.17\end{pmatrix},\begin{pmatrix}\bar{\Delta}p_1\\ \bar{\Delta}p_2\\ \bar{\Delta}p_3\end{pmatrix}=\begin{pmatrix}0\\ -0.002\\ -0.005\end{pmatrix},$$

$$\begin{pmatrix}\bar{\Delta}w_1\\ \bar{\Delta}w_2\\ \bar{\Delta}w_3\end{pmatrix}=\begin{pmatrix}0.09\\ 0.08\\ 0.07\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}R_1\\ \bar{\Delta}R_2\\ \bar{\Delta}R_3\end{pmatrix}=\begin{pmatrix}-2.36\\ -2.79\\ 7\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}\theta_1\\ \bar{\Delta}\theta_2\\ \bar{\Delta}\theta_3\end{pmatrix}=\begin{pmatrix}-0.12\\ -0.14\\ 0.54\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}\bar{k}_1\\ \bar{\Delta}\bar{k}_1\\ \bar{\Delta}\bar{k}_1\end{pmatrix}=\begin{pmatrix}0.01\\ 0\\ -0.01\end{pmatrix},$$

$$\begin{pmatrix}\bar{\Delta}l_1\\ \bar{\Delta}l_2\\ \bar{\Delta}l_3\end{pmatrix}=\begin{pmatrix}2.42\\ 2.97\\ -6.55\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}c_{1i}\\ \bar{\Delta}c_{2i}\\ \bar{\Delta}c_{3i}\end{pmatrix}=\begin{pmatrix}0.005\\ -0.004\\ -0.01\end{pmatrix},\ \begin{pmatrix}\bar{\Delta}c_{1s}\\ \bar{\Delta}c_{2s}\\ \bar{\Delta}c_{3s}\end{pmatrix}=\begin{pmatrix}0.01\\ 0\\ -0.01\end{pmatrix}. \quad (29)$$

As region 3 becomes more attractive (with all the other variables fixed) for living, the region has more people. Due to the competition, the governments reduce the tax rates. The enlarged residential area attracts more people to region 3. The increased population is associated with increases in region 3's expenditures on public goods. At the new equilibrium, region 3's lot size per household is reduced. As more people move to the lest productive region, the nation has lower income and less capital stocks. The rate of interest is reduced. The wage rates are enhanced. The prices of services are reduced slightly. Region 3's household enjoys higher amenity but pays higher rent and lives in smaller house. Region 3's other real macro-economic variables are augmented, while the corresponding variables of the other two regions are reduced.

4.5. Region 1's total factor productivity of the industrial sector is augmented

We study how the equilibrium structure is affected by the following rise in region 1's total factor productivity of the industrial sector: $A_{1i} = 1.3 \text{ to } 1.31.$ We list the simulation result in (30):

$$\bar{\Delta}Y = 3.5,\ \bar{\Delta}K = 3.84,\ \bar{\Delta}r = 1.6,$$

$$\begin{pmatrix} \bar{\Delta}\tau_{1k} \\ \bar{\Delta}\tau_{2k} \\ \bar{\Delta}\tau_{3k} \end{pmatrix} = \begin{pmatrix} \bar{\Delta}\tau_{1w} \\ \bar{\Delta}\tau_{2w} \\ \bar{\Delta}\tau_{3w} \end{pmatrix} = \begin{pmatrix} 0.49 \\ 0.4 \\ 0.36 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{Y}_1 \\ \bar{\Delta}\bar{Y}_2 \\ \bar{\Delta}\bar{Y}_3 \end{pmatrix} = \begin{pmatrix} 24.03 \\ -27 \\ -25.12 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}Y_1 \\ \bar{\Delta}Y_2 \\ \bar{\Delta}Y_3 \end{pmatrix} = \begin{pmatrix} 23.38 \\ -27.38 \\ -25.51 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_1 \\ \bar{\Delta}N_2 \\ \bar{\Delta}N_3 \end{pmatrix} = \begin{pmatrix} 22.7 \\ -27 \\ -25.16 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_1 \\ \bar{\Delta}K_2 \\ \bar{\Delta}K_3 \end{pmatrix} = \begin{pmatrix} 21.94 \\ -28.18 \\ -26.28 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{K}_1 \\ \bar{\Delta}\bar{K}_2 \\ \bar{\Delta}\bar{K}_3 \end{pmatrix} = \begin{pmatrix} 24.04 \\ -27 \\ -25.12 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}F_{1i} \\ \bar{\Delta}F_{2i} \\ \bar{\Delta}F_{3i} \end{pmatrix} = \begin{pmatrix} 23.24 \\ -27.47 \\ -25.6 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}F_{1s} \\ \bar{\Delta}F_{2s} \\ \bar{\Delta}F_{3s} \end{pmatrix} = \begin{pmatrix} 23.09 \\ -27.01 \\ -25.14 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}N_{1i} \\ \bar{\Delta}N_{2i} \\ \bar{\Delta}N_{3i} \end{pmatrix} = \begin{pmatrix} 22.53 \\ -27.11 \\ -25.26 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_{1s} \\ \bar{\Delta}N_{2s} \\ \bar{\Delta}N_{3s} \end{pmatrix} = \begin{pmatrix} 23.33 \\ -26.63 \\ -24.78 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_{1i} \\ \bar{\Delta}K_{2i} \\ \bar{\Delta}K_{3i} \end{pmatrix} = \begin{pmatrix} 21.79 \\ -28.27 \\ -26.37 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_{1s} \\ \bar{\Delta}K_{2s} \\ \bar{\Delta}K_{3s} \end{pmatrix} = \begin{pmatrix} 22.58 \\ -27.8 \\ -25.89 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}p_1 \\ \bar{\Delta}p_2 \\ \bar{\Delta}p_3 \end{pmatrix} = \begin{pmatrix} 0.77 \\ 0.02 \\ 0.03 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}w_1 \\ \bar{\Delta}w_2 \\ \bar{\Delta}w_3 \end{pmatrix} = \begin{pmatrix} 0.57 \\ -0.5 \\ -0.45 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}R_1 \\ \bar{\Delta}R_2 \\ \bar{\Delta}R_3 \end{pmatrix} = \begin{pmatrix} 24.04 \\ -27 \\ -25.12 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}\theta_1 \\ \bar{\Delta}\theta_2 \\ \bar{\Delta}\theta_3 \end{pmatrix} = \begin{pmatrix} 1.14 \\ -1.56 \\ -1.43 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \end{pmatrix} = \begin{pmatrix} 1.11 \\ 0.03 \\ 0.08 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}l_1 \\ \bar{\Delta}l_2 \\ \bar{\Delta}l_3 \end{pmatrix} = \begin{pmatrix} -18.49 \\ 37.03 \\ 33.65 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}c_{1i} \\ \bar{\Delta}c_{2i} \\ \bar{\Delta}c_{3i} \end{pmatrix} = \begin{pmatrix} 1.11 \\ 0.03 \\ 0.08 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}c_{1s} \\ \bar{\Delta}c_{2s} \\ \bar{\Delta}c_{3s} \end{pmatrix} = \begin{pmatrix} 0.33 \\ 0.01 \\ 0.05 \end{pmatrix}. \quad (30)$$

The national output and capital stocks are increased. The rate of interest is increased. The government raise the tax rates. More people work in region 1. The region's wage rate is increased. Region's lot size is reduced. The region's rent is increased. The region's total tax income is increased. The amenity level is enhanced. Due to the reduced tax incomes, the two other regions provide less public goods, which reduce the regional attractiveness. It should be mentioned that there are different approaches to explaining wage disparities between regions (e.g., Glaeser and Maré 2001; Duranton and Monastiriotis 2002; Combes *et al* 2003; Rey and Janikas 2005; Candelaria *et.al.* 2015). The previous studies identify many factors, such as spatial differences in education opportunities, innovation and knowledge diffusion, skill composition of the workforce, local interactions, regional amenities, as well as non-human endowments. Our simulation result shows that technological change in one region may encourage wage disparity. This also implies that if technological differences between regions are not large, wage rates may tend to converge if the other factors weakly affect the differences.

4.6. Region 1's total factor productivity of the services sector is augmented

We study how the equilibrium structure is affected by the following rise in region 1's total factor productivity of the service sector: $A_{1s} = 12 \text{ to } 12.1$. We list the simulation result in (31):

$$\bar{\Delta}Y = 0.15, \bar{\Delta}K = 0.16, \bar{\Delta}r = 0.09,$$

$$\begin{pmatrix} \bar{\Delta}\tau_{1k} \\ \bar{\Delta}\tau_{2k} \\ \bar{\Delta}\tau_{3k} \end{pmatrix} = \begin{pmatrix} \bar{\Delta}\tau_{1w} \\ \bar{\Delta}\tau_{2w} \\ \bar{\Delta}\tau_{3w} \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.02 \\ 0.02 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{Y}_1 \\ \bar{\Delta}\bar{Y}_2 \\ \bar{\Delta}\bar{Y}_3 \end{pmatrix} = \begin{pmatrix} 1.24 \\ -1.5 \\ -1.36 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}Y_1 \\ \bar{\Delta}Y_2 \\ \bar{\Delta}Y_3 \end{pmatrix} = \begin{pmatrix} 1.21 \\ -1.53 \\ -1.39 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_1 \\ \bar{\Delta}N_2 \\ \bar{\Delta}N_3 \end{pmatrix} = \begin{pmatrix} 1.24 \\ 1.5 \\ -1.36 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_1 \\ \bar{\Delta}K_2 \\ \bar{\Delta}K_3 \end{pmatrix} = \begin{pmatrix} 1.14 \\ -1.59 \\ -1.44 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{K}_1 \\ \bar{\Delta}\bar{K}_2 \\ \bar{\Delta}\bar{K}_3 \end{pmatrix} = \begin{pmatrix} 1.24 \\ -1.5 \\ -1.36 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}F_{1i} \\ \bar{\Delta}F_{2i} \\ \bar{\Delta}F_{3i} \end{pmatrix} = \begin{pmatrix} 1.2 \\ -1.53 \\ -1.39 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}F_{1s} \\ \bar{\Delta}F_{2s} \\ \bar{\Delta}F_{3s} \end{pmatrix} = \begin{pmatrix} 2.08 \\ -1.5 \\ -1.36 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}N_{1i} \\ \bar{\Delta}N_{2i} \\ \bar{\Delta}N_{3i} \end{pmatrix} = \begin{pmatrix} 1.24 \\ -1.51 \\ -1.37 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_{1s} \\ \bar{\Delta}N_{2s} \\ \bar{\Delta}N_{3s} \end{pmatrix} = \begin{pmatrix} 1.27 \\ -1.47 \\ -1.33 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_{1i} \\ \bar{\Delta}K_{2i} \\ \bar{\Delta}K_{3i} \end{pmatrix} = \begin{pmatrix} 1.14 \\ -1.6 \\ -1.45 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}K_{1s} \\ \bar{\Delta}K_{2s} \\ \bar{\Delta}K_{3s} \end{pmatrix} = \begin{pmatrix} 1.17 \\ -1.56 \\ -1.42 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}p_1 \\ \bar{\Delta}p_2 \\ \bar{\Delta}p_3 \end{pmatrix} = \begin{pmatrix} -0.83 \\ 0.001 \\ 0.002 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}w_1 \\ \bar{\Delta}w_2 \\ \bar{\Delta}w_3 \end{pmatrix} = \begin{pmatrix} -0.03 \\ -0.03 \\ -0.02 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}R_1 \\ \bar{\Delta}R_2 \\ \bar{\Delta}R_3 \end{pmatrix} = \begin{pmatrix} 1.24 \\ -1.5 \\ -1.36 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}\theta_1 \\ \bar{\Delta}\theta_2 \\ \bar{\Delta}\theta_3 \end{pmatrix} = \begin{pmatrix} 0.06 \\ -0.08 \\ -0.07 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \end{pmatrix} = \begin{pmatrix} -0.002 \\ 0.002 \\ 0.004 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}l_1 \\ \bar{\Delta}l_2 \\ \bar{\Delta}l_3 \end{pmatrix} = \begin{pmatrix} -1.23 \\ 1.52 \\ 1.38 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}c_{1i} \\ \bar{\Delta}c_{2i} \\ \bar{\Delta}c_{3i} \end{pmatrix} = \begin{pmatrix} -0.002 \\ 0.001 \\ 0.004 \end{pmatrix}, \begin{pmatrix} \bar{\Delta}c_{1s} \\ \bar{\Delta}c_{2s} \\ \bar{\Delta}c_{3s} \end{pmatrix} = \begin{pmatrix} 0.83 \\ 0.001 \\ 0.002 \end{pmatrix}. \quad (31)$$

As in the previous case, the national output and capital stocks are increased, and the rate of interest is increased. The government raise the tax rates. More people work in region 1. The wage rates are reduced.

4.7. The propensity to consume housing is enhanced

We deal with how the equilibrium structure is affected by the following increase in the population's propensity to consume housing: $\eta_0 = 0.07 \text{ to } 0.072$. We list the simulation result in (32):

$$\bar{\Delta}Y = -1.05, \ \bar{\Delta}K = -1.71, \ \bar{\Delta}r = 0.32,$$

$$\begin{pmatrix} \bar{\Delta}\tau_{1k} \\ \bar{\Delta}\tau_{2k} \\ \bar{\Delta}\tau_{3k} \end{pmatrix} = \begin{pmatrix} \bar{\Delta}\tau_{1w} \\ \bar{\Delta}\tau_{2w} \\ \bar{\Delta}\tau_{3w} \end{pmatrix} = \begin{pmatrix} -0.55 \\ -0.56 \\ -0.57 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}\bar{Y}_1 \\ \bar{\Delta}\bar{Y}_2 \\ \bar{\Delta}\bar{Y}_3 \end{pmatrix} = \begin{pmatrix} -7.41 \\ 8.17 \\ 6.29 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}Y_1 \\ \bar{\Delta}Y_2 \\ \bar{\Delta}Y_3 \end{pmatrix} = \begin{pmatrix} -6.91 \\ 8.76 \\ 6.86 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_1 \\ \bar{\Delta}N_2 \\ \bar{\Delta}N_3 \end{pmatrix} = \begin{pmatrix} -6.8 \\ 8.87 \\ 6.96 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}K_1 \\ \bar{\Delta}K_2 \\ \bar{\Delta}K_3 \end{pmatrix} = \begin{pmatrix} -7.13 \\ 8.51 \\ 6.63 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}\bar{K}_1 \\ \bar{\Delta}\bar{K}_2 \\ \bar{\Delta}\bar{K}_3 \end{pmatrix} = \begin{pmatrix} -7.69 \\ 7.96 \\ 6.07 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}F_{1i} \\ \bar{\Delta}F_{2i} \\ \bar{\Delta}F_{3i} \end{pmatrix} = \begin{pmatrix} -6.75 \\ 8.94 \\ 7.04 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}F_{1s} \\ \bar{\Delta}F_{2s} \\ \bar{\Delta}F_{3s} \end{pmatrix} = \begin{pmatrix} -7.59 \\ 7.95 \\ 6.07 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}N_{1i} \\ \bar{\Delta}N_{2i} \\ \bar{\Delta}N_{3i} \end{pmatrix} = \begin{pmatrix} -6.65 \\ 9.05 \\ 7.13 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_{1s} \\ \bar{\Delta}N_{2s} \\ \bar{\Delta}N_{3s} \end{pmatrix} = \begin{pmatrix} -7.49 \\ 8.06 \\ 6.17 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}K_{1i} \\ \bar{\Delta}K_{2i} \\ \bar{\Delta}K_{3i} \end{pmatrix} = \begin{pmatrix} -6.97 \\ 8.7 \\ 6.81 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}K_{1s} \\ \bar{\Delta}K_{2s} \\ \bar{\Delta}K_{3s} \end{pmatrix} = \begin{pmatrix} -7.81 \\ 7.72 \\ 5.85 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}p_1 \\ \bar{\Delta}p_2 \\ \bar{\Delta}p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.003 \\ 0.006 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}w_1 \\ \bar{\Delta}w_2 \\ \bar{\Delta}w_3 \end{pmatrix} = \begin{pmatrix} -0.11 \\ -0.01 \\ -0.09 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}R_1 \\ \bar{\Delta}R_2 \\ \bar{\Delta}R_3 \end{pmatrix} = \begin{pmatrix} -4.95 \\ 11.04 \\ 9.1 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}\theta_1 \\ \bar{\Delta}\theta_2 \\ \bar{\Delta}\theta_3 \end{pmatrix} = \begin{pmatrix} -0.42 \\ 0.36 \\ 0.27 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \end{pmatrix} = \begin{pmatrix} -0.85 \\ -0.84 \\ -0.82 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}l_1 \\ \bar{\Delta}l_2 \\ \bar{\Delta}l_3 \end{pmatrix} = \begin{pmatrix} 7.3 \\ -8.14 \\ -6.51 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}c_{1i} \\ \bar{\Delta}c_{2i} \\ \bar{\Delta}c_{3i} \end{pmatrix} = \begin{pmatrix} -0.85 \\ -0.84 \\ -0.82 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}c_{1s} \\ \bar{\Delta}c_{2s} \\ \bar{\Delta}c_{3s} \end{pmatrix} = \begin{pmatrix} -0.85 \\ -0.84 \\ -0.82 \end{pmatrix}. \quad (32)$$

As the propensity to consume housing is increased, some people move away from region 1 to the other two regions as region 1's land is relatively fully occupied. The reduced labor force in region 1 leads to national economic decline in terms of output and wealth (excluding land). Region 1 has less tax income and lower amenity, while the other two regions experience the opposite changes. All the households have less wealth and lower levels of consumption of goods and services. Region 1's lot size rises, but the other two regions' lot size is reduced. The wage rates are reduced. The prices of services change slightly. The values of region 1's real macro-economic variables are lowered, while the corresponding variables of the two regions are enhanced.

4.8. The propensity to consume housing is enhanced

We analyze how the equilibrium structure is affected by the following increase in the population's propensity to save: $\lambda_0 = 0.075 \text{ to } 0.0752$. We list the simulation result in (33):

$$\bar{\Delta} Y = 0.32, \ \bar{\Delta} K = 0.59, \ \bar{\Delta} r = -0.22,$$

$$\begin{pmatrix} \bar{\Delta}\tau_{1k} \\ \bar{\Delta}\tau_{2k} \\ \bar{\Delta}\tau_{3k} \end{pmatrix} = \begin{pmatrix} \bar{\Delta}\tau_{1w} \\ \bar{\Delta}\tau_{2w} \\ \bar{\Delta}\tau_{3w} \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.02 \\ 0.03 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}\bar{Y}_1 \\ \bar{\Delta}\bar{Y}_2 \\ \bar{\Delta}\bar{Y}_3 \end{pmatrix} = \begin{pmatrix} 1.56 \\ -0.51 \\ -2{,}5 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}Y_1 \\ \bar{\Delta}Y_2 \\ \bar{\Delta}Y_3 \end{pmatrix} = \begin{pmatrix} 1.56 \\ -0.51 \\ -2.5 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_1 \\ \bar{\Delta}N_2 \\ \bar{\Delta}N_3 \end{pmatrix} = \begin{pmatrix} 1.48 \\ -0.58 \\ -2.56 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}K_1 \\ \bar{\Delta}K_2 \\ \bar{\Delta}K_3 \end{pmatrix} = \begin{pmatrix} 1.72 \\ -0.36 \\ -2.36 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}\bar{K}_1 \\ \bar{\Delta}\bar{K}_2 \\ \bar{\Delta}\bar{K}_3 \end{pmatrix} = \begin{pmatrix} 1.85 \\ -0.23 \\ -2.22 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}F_{1i} \\ \bar{\Delta}F_{2i} \\ \bar{\Delta}F_{3i} \end{pmatrix} = \begin{pmatrix} 1.55 \\ -0.52 \\ -2.51 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}F_{1s} \\ \bar{\Delta}F_{2s} \\ \bar{\Delta}F_{3s} \end{pmatrix} = \begin{pmatrix} 1.58 \\ -0.49 \\ -2.48 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}N_{1i} \\ \bar{\Delta}N_{2i} \\ \bar{\Delta}N_{3i} \end{pmatrix} = \begin{pmatrix} 1.47 \\ -0.59 \\ -2.57 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}N_{1s} \\ \bar{\Delta}N_{2s} \\ \bar{\Delta}N_{3s} \end{pmatrix} = \begin{pmatrix} 1.5 \\ -0.56 \\ -0.54 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}K_{1i} \\ \bar{\Delta}K_{2i} \\ \bar{\Delta}K_{3i} \end{pmatrix} = \begin{pmatrix} 1.72 \\ -0.36 \\ -2.36 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}K_{1s} \\ \bar{\Delta}K_{2s} \\ \bar{\Delta}K_{3s} \end{pmatrix} = \begin{pmatrix} 1.74 \\ -0.34 \\ -2.34 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}p_1 \\ \bar{\Delta}p_2 \\ \bar{\Delta}p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -0.002 \\ -0.004 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}w_1 \\ \bar{\Delta}w_2 \\ \bar{\Delta}w_3 \end{pmatrix} = \begin{pmatrix} 0.08 \\ 0.07 \\ 0.06 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}R_1 \\ \bar{\Delta}R_2 \\ \bar{\Delta}R_3 \end{pmatrix} = \begin{pmatrix} 1.58 \\ -0.49 \\ -2.48 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}\theta_1 \\ \bar{\Delta}\theta_2 \\ \bar{\Delta}\theta_3 \end{pmatrix} = \begin{pmatrix} 0.08 \\ -0.02 \\ -0.12 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \\ \bar{\Delta}\bar{k}_1 \end{pmatrix} = \begin{pmatrix} 0.37 \\ 0.36 \\ 0.35 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}l_1 \\ \bar{\Delta}l_2 \\ \bar{\Delta}l_3 \end{pmatrix} = \begin{pmatrix} -1.46 \\ 0.59 \\ 2.63 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta}c_{1i} \\ \bar{\Delta}c_{2i} \\ \bar{\Delta}c_{3i} \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.09 \\ 0.09 \end{pmatrix}, \ \begin{pmatrix} \bar{\Delta}c_{1s} \\ \bar{\Delta}c_{2s} \\ \bar{\Delta}c_{3s} \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.09 \\ 0.09 \end{pmatrix}. \quad (33)$$

As the propensity to save is increased, some people move to region 1 from the other two regions. The increase of labor force in region 1 leads to national economic development in terms of output and wealth (excluding land). Region 1 has more tax income and higher amenity, while the other two regions experience the opposite changes. All the households have more wealth and higher levels of consumption of goods and services. Region 1's lot size falls, but the other two regions' lot size is augmented. The wage rates are enhanced. The prices of services change slightly. The values of region 1's real macro-economic variables are enhanced, while the corresponding variables of the two regions are decreased.

5. Conclusions

This paper studied an inter-regional economic growth with regional governments' competition in taxation. We built the model by extending Uzawa's two-sector growth model to any number of regions. The model treats wealth/capital accumulation, economic structures, factor distributions, interregional population distribution, amenity, regions' tax rates as endogenous variables. The model is built on micro-economic foundation in the sense that firms' behavior is described by profit maximization, households' behavior by utility maximization, markets by perfect competition, and regional governments' behavior by choosing tax rates to maximize utility. We identified the existence of an equilibrium point and carried our comparative analysis with regards to different parameters. As the model is structurally general, it is possible to deal with various national as well as regional growth issues. It is straightforward to analyze behavior of the model with other forms of production or utility functions. We may also introduce heterogeneous households and imperfect competition to the system. There are many other issues, such as national and regional debts, related to tax competition between regions.

Appendix: Proving the Lemma

We now show a procedure to determine the dynamics of the system in two differential equations with general production functions. First, from equations (2) we obtain:

$$z_j \equiv \frac{r + \delta_k}{w_j} = \frac{a_j N_{ji}}{K_{ji}} = \frac{b_j N_{js}}{K_{js}}, \quad (A1)$$

where

$$a_j \equiv \frac{\alpha_{ji}}{\beta_{ji}}, \quad b_j \equiv \frac{\alpha_{js}}{\beta_{js}}.$$

Insert $z_j / a_j \equiv N_{ji}/K_{ji}$ in $r + \delta_{kj} = \alpha_{ji} F_{ji}/K_{ji}$ from (1):

$$r(z_j) = \frac{\alpha_{ji} A_{ji}}{a_j^{\beta_{ji}}} z_j^{\beta_{ji}} - \delta_{kj}, \quad j = 1, \ldots, J. \ (A2)$$

From (A2) we get:

$$z_j(z_1) = a_j \left(\frac{r + \delta_{kj}}{\alpha_{ji} A_{ji}} \right)^{1/\beta_{ji}}, \quad j = 2, \ldots, J. \ (A3)$$

From (A1) and (A2), we have:

$$w_j(z_1) = \frac{r + \delta_k}{z_j}. \text{ (A4)}$$

From $z_j = b_j N_{js}/K_{js}$ and (1), we have:

$$p_j(z_1) = \frac{b_j^{\beta_{js}}(r + \delta_k)}{\alpha_{js} A_{js} z_j^{\beta_{js}}}. \ (A5)$$

From (11) and $p_j c_{js} = \gamma \hat{y}_j$ we have:

$$\gamma \hat{y}_j N_j = p_j F_{js}. \qquad \text{(A6)}$$

Insert (1) in (A6):

$$\gamma \hat{y}_j N_j = \frac{w_j N_{js}}{\beta_{js}}. \ (A7)$$

Insert (23) in (21):

$$\hat{y}_j = \frac{\tilde{\rho}\, h_j(t)}{\tilde{\rho} + \bar{d}}. \ (A8)$$

From (9) and (14). We have:

$$R_j = \frac{\eta \hat{y}_j N_1}{L_1},$$

Apply $U_j = U_q$ to (20):

$$\bar{\theta}_j L_j^{\eta_0} N_j^{d+\bar{d}-\eta_0} Y_j^{0\bar{d}} p_j^{-\gamma_0} \tau_j^{\bar{d}} \hat{y}_j^{\tilde{\rho}} = \bar{\theta}_q L_q^{\eta_0} N_q^{d+\bar{d}-\eta_0} Y_q^{0\bar{d}} p_q^{-\gamma_0} \tau_q^{\bar{d}} \hat{y}_q^{\tilde{\rho}}. \ (A9)$$

Solve (A9)

$N_j = \Lambda_j N_1$, j = 2, …, J, (A10)

where

$$\Lambda_j\left(z_1,(\bar{k}_q)\right) \equiv \left(\frac{\bar{\theta}_1\, L_1^{\eta_0}\, Y_1^{0\bar{d}}\, p_1^{-\gamma_0}\, \tau_1^{\bar{d}}\, \hat{y}_1^{\tilde{\rho}}}{\bar{\theta}_j\, L_j^{\eta_0}\, Y_j^{0\bar{d}}\, p_j^{-\gamma_0}\, \tau_j^{\bar{d}}\, \hat{y}_j^{\tilde{\rho}}}\right)^{1/(d+\bar{d}-\eta_0)}.$$

Insert (A10) in (16):

$$N_1\left(z_1,(\bar{k}_q)\right) = \frac{N}{\sum_{j=1}^{J}\Lambda_j}, \quad \Lambda_1 = 1. \ (A11)$$

With (A10) and (A11) we determine the population distribution as functions of z_1 and $(\bar{k}_q)$. By $l_j\, R_j = \eta\, \hat{y}_j$ and $l_j\, N_j = L_j$, we have:

$$R_j\left(z_1,(\bar{k}_q)\right) = \frac{\eta\, \hat{y}_j\, N_j}{L_j}. \ \text{(A12)}$$

From (A7) we solve:

$$N_{js} = \frac{\beta_{js}\gamma\, \hat{y}_j\, N_j}{w_j}. \ (A13)$$

With $N_{ji} + N_{js} = N_j$ and (A13), we get:

$$N_{ji}\left(z_1,(\bar{k}_q)\right) = N_j - N_{js}, j = 1,\dots,J. \ (A14)$$

Equation (11) Implies:

$$\sum_{j=1}^{J}\left(K_{ji} + K_{js}\right) = \sum_{j=1}^{J}\bar{k}_j\, N_j. \ (A15)$$

Insert (A1) in (A15):

$$\Phi\left(z_1,(\bar{k}_j)\right) \equiv \sum_{j=1}^{J}\left(\frac{a_j N_{ji} + b_j N_{js}}{z_j}\right) - \sum_{j=1}^{J}\bar{k}_j\, N_j = 0. \ (A16)$$

Substituting $s_j = \lambda\, \hat{y}_j$ into (10) yields:

$$\dot{\bar{k}}_j = \Phi_j(z_1, \bar{k}_j) \equiv \lambda\,\hat{y}_j - \bar{k}_j. \ (A17)$$

By (A17), we have at equilibrium

$$\lambda\,\hat{y}_j = \bar{k}_j. \ (A18)$$

By (A6) and (A18), we have $J+1$ equations to determine $J+1$ variables, z_1 and $(\bar{k}_j)$. We can thus determine the equilibrium point.

References:

Ahlfeldt, G. (2011) ”If Alonso Was Right: Modeling Accessibility and Explaining the Residential Land Gradient”, *Journal of Regional Science*, 51, 318-38.

Almazan, A., DeMotta, A., Titman, S. (2007) ”Firm Location and the Creation and Utilization of Human Capital”, *Review of Economic Studies*, 74, 1305–27.

Andersson, F. and Forslid, R. (2003) ”Tax Competition and Economic Geography”, *Journal of Public Economic Theory*, 5, 279-303.

Bairoch, P. (1993) *Economics and World History: Myths and Paradoxes*, Chicago: University of Chicago Press.

Baldwin, R.E. and Krugman, P. (2004) ”Agglomeration, Integration and Tax Harmonization”, *European Economic Review*, 48, 1-23.

Bayindir-Upmann, T. and Ziad, A. (2005) ”Existence of Equilibria in a Basic Tax Competition Model”, Regional Science and Urban Economics, 35, 1-22.

Bertoletti, P. and Etro, F. (2015) ”Monopolistic Competition When Income Matters”, *The Economic Journal*, 127(603), 1217-43.

Borck, R. and Pflüger, M. (2006) ”Agglomeration and Tax Competition”, *European Economic Review* (in Press).

Brecher, R.A., Chen, Z.Q. and Choudhri, E.U. (2002) ”Absolute and Comparative Advantage, Reconsidered: The Pattern of International Trade with Optimal Saving”, *Review of International Economics*, 10, 645-656.

Blomquist, G.C., Berger, M.C., and Hoehn, J.C. (1988) ”New Estimates of Quality of Life in Urban Areas”, *American Economic Review*, 78, 89-107.

Can, A. and Megbolugbe, I. (1997) ”Spatial Dependence and House Price Index Construction”, The Journal of Real Estate Finance and Economics, 14, 203-22.

Candelaria, C., Daly, M., and Hale, G. (2015) ”Persistence of Regional Wage Differences in China”, *Pacific Economic Review*, 20(3), 365-87.

Charlot, S., Gaigné, C., Robert-Nicoud, F., Thisse, J.-F. (2006) ”Agglomeration and Welfare: The Core–Periphery Model In the Light of Bentham, Kaldor, and Rawls”, *Journal of Public Economics*, 90, 325–47.

Chasco, C. and LeGallo, J. (2011) ”The Impact of Objective and Subjective Measures of Air Quality and Noise on House Prices: A Multilevel Approach for Downtown Madrid”, *European Regional Science Association, ERSA conference papers with number* ersa11p168.

Chen, Y., Irwin, E.G., and Jayaprakash, C. (2013) "Population Dispersion vs. Concentration in a Two-Region Migration Model with Endogenous Natural Amenities", *Journal of Regional Science*, 53, 256–73.

Combes, P.G., Duraton, G. and Gobillon, L. (2004) "Spatial Wage Disparities: Sorting Matters!" *CEPR Discussion Paper*, 4220.

Diamond, D.B. and Tolley, G.S. (Eds.)(1981) *The Economics of Urban Amenities*, New York: Academic Press.

Dubin, R. (1992) "Spatial Autocorrelation and Neighborhood Quality", *Regional Science and Urban Economics*, 22, 433-52.

Duranton, G. and Monastiriotis, V. (2002) "Mind the Gaps: The Evolution of Regional Earnings Inequalities in the UK 1982-1997", *Journal of Regional Science*, 42, 219-256.

Forslid, R. and Ottaviano, G.I.P. (2003) "Trade and Location: Two Analytically Solvable Models", *Journal of Economic Geography*, 3, 229–40.

Fujita, M., Krugman, P. and Venables, A. (1999) *The Spatial Economy*, Cambridge MA: MIT Press.

Glaeser, E.L. and Maré, D.C. (2001) "Cities and Skills", *Journal of Labor Economics*, 19, 316-42.

Graves, P. (1979) "A Life-Cycle Empirical Analysis of Migration and Climate by Race", *Journal of Urban Economics*, 6, 135-47.

Helbich, M., Brunauer, W., and Nijkamp, P. (2014) Spatial Heterogeneity in Hedonic House Price Models: The Case of Austria. Urban Studies 51, 390-411.

Hirschman, A.O. (1958) *The Strategy of Economic Development*, New Haven: Yale University Press.

Ihori, T. and Yang, C.C. (2009) "Interregional Tax Competition and Intraregional Political Competition: The Optimal Provision of Public Goods under Representative Democracy", *Journal of Urban Economics*, 66(3), 210-17.

Kanemoto, Y. (1980) *Theories of Urban Externalities*, Amsterdam: North-Holland.

Krugman, P. (1991) "Increasing Returns and Economic Geography", *Journal of Political Economy*, 99, 483-499.

Krugman, P. (1993) "On the Number and Location of Cities", *European Economic Review*, 37, 293–98.

Kuznets, S. (1966) *Modern Economic Growth, Rate Structure, and Spread*, New Haven: Yale University Press.

Liu, C.H., Rosenthal, S.S. and Strange, W.C. (2018) "The Vertical City: Rent Gradients, Spatial Structure, and Agglomeration Economies", *Journal of Urban Economics*, 106 (July), 101-122.

Malpezzi, S. (2003) "Hedonic Pricing Models: A Selective and Applied Review", In T. O'Sullivan and K. Gibb (Eds.) *Housing Economics and Public Policy*, Oxford: Blackwell, pp. 67-89.

McMillen, D.P. (2010) "Issues in Spatial Data Analysis", *Journal of Regional Science*, 50, 119-41.

Myrdal, G. (1957) *Economic Theory and Underdeveloped Regions*, London: Duckworth.

Nocco, A., Ottaviano, G.L.P., and Salto, M. (2017) "Monopolistic Competition and Optimum Product Selection: Why and How Heterogeneity Matters", *Research in Economics*, 71(4), 704-17.

Oniki, H. and Uzawa, H. (1965) ”Patterns of Trade and Investment in a Dynamic Model of International Trade”, *Review of Economic Studies,* 32, 15-38.

Parenti, M., Ushchev, P., and Thisse, J.F. (2017) ”Toward a Theory of Monopolistic Competition”, *Journal of Economic Theory,* 167(January), 86-115.

Partridge, M. D., Rickman, D.S., Ali, K., and Olfert, M.R. (2008) ”Lost in Space: Population Dynamics in the American Hinterlands and Small Cities”, *Journal of Economic Geography,* 8, 727–57.

Picard, P.M. and Zenou, Y. (2018) ”Urban Spatial Structure, Employment and Social Ties”, *Journal of Urban Economics,* 104 (May), 77-93.

Rey, S.J. and Janikas, M.V. (2005) ”Regional Convergence, Inequality and Space”, *Journal of Economic Geography,* 5, 155-76.

Roback, J. (1982) ”Wages, Rents, and the Quality of Life”, Journal of Political Economy, 90, 1257-78.

Rosen, S. (1974) ”Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition”, *Journal of Political Economy,* 82, 34-55.

Saez, E. and Stantcheva, S. (2018) ”A Simpler Theory of Optimal Capital Taxation”, *Journal of Public Economics,* 162, 120-42.

Sheppard, S. (1997) ”Hedonic Analysis of Housing Markets”, In: P. Chesire, E. Mills (Eds.) *Handbook of Regional and Urban Economics, vol. 3,* Amsterdam: Elsevier, 1595-635..

Sorger, G. (2002) ”On the Multi-Region Version of the Solow-Swan Model”, *The Japanese Economic Review,* 54, 146-164.

Tabuchi, T. (2014) ”Historical Trends of Agglomeration to the Capital Region and New Economic Geography”, *Regional Science and Urban Economics,* 44, 50–59.

Tivadar, M. and Jayet, H. (2019) ”Endogenous Generation of Amenities and the Dynamics of City Structure”, *Regional Science and Urban Economics,* 74, 49-56.

Uzawa, H. (1961) ”On a Two-Sector Model of Economic Growth”, *Review of Economic Studies*, 29, 47-70.

Wildasin, D.E. (1988) ”Nash Equilibria in Models of Fiscal Competition”, *Journal of Public Economics,* 35, 229-40.

Wildasin, D.E. (1991) ”Some Rudimentary Duopolity Theory”, *Regional Science and Urban Economics,* 21, 393-421.

Wilson, J.D. (1986) ”A Theory of Interregional Tax Competition”, *Journal of Urban Economics,* 19, 296-315.

Wilson, J.D. (1991) ”Tax Competition with Interregional Differences in Factor Endowments”, *Regional Science and Urban Economics,* 21, 423-451.

Zhang, W.B. (1993) ”Wages, Service Prices and Rent - Urban Division of Labor and Amenities”, *Seoul Journal of Economics,* 6, 97-113.

Zhang, W.B. (1996) ”Economic Geography with two Regions: Capital Accumulation and Economic Structure”, *Australian Economic Papers,* 35, 225-35.

Zhang, W.B. (2005) *Economic Growth Theory,* London: Ashgate.

Zhang, W.B. (2009) ”Agglomeration and Returns to Scale with Capital and Public Goods in a Multi-Regional Economy”, *International Economic Journal,* 23, 81-109

Zhang, W.B. (2008) *International Trade Theory: Capital, Knowledge, Economic Structure, Money and Prices over Time and Space,* Berlin: Springer.

Zhang, W.B. (2018) ”Regional Economic Structure, Amenities and Disparities in a Neoclassical Growth Model”, *Eastern European Journal for Regional Studies*, 4(2), 73-93.

Zodrow, G.R. and Mieszkowski, P. (1986) ”Pigou, Tiebout, Property Taxation, and the Underprovision of Local Public Goods”, *Journal of Urban Economics*, 19, 356-70.

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